LOYOLA UNIVERSITY CHICAGO
AD • MAJOREM • DEI • GLORIAM 1870

Department of Mathematics and Statistics
Lake shore campus | 1032 W. Sheridan Road | Chicago, Illinois 60660
Phone 773.508.3558 | Fax 773.508.2123 | www.math.luc.edu

Math 131 - Spring 2023 - Final Exam

Print name:_____

Section number:_____ **Instructor's name:**_____

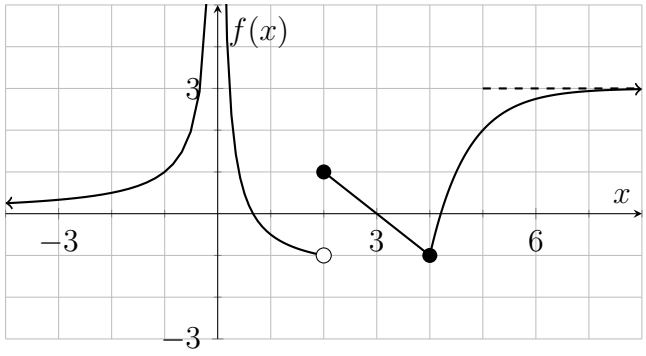
Directions:

- This exam has 12 questions.
- You will have two hours to complete this exam.
- It will be graded out of 106 points.
- Show your work. Answers (even correct ones) without the corresponding work will receive no credit.
- A formula sheet has been provided on the last page of this exam. You may not refer to any other notes or textbooks during the exam.
- You may use a calculator as long as it is unable to connect to the internet.
- You may only speak with your instructor during the exam.

Good luck!

Question:	1	2	3	4	5	6	7	8	9	10	11	12	Total
Points:	12	6	6	6	14	10	4	10	10	10	10	8	106
Score:													

1. [12 points] Using the graph below, find each of the following. If the answer does not exist, write "DNE". The dotted line on the right of the graph represents a horizontal asymptopte and is not a part of the function.



(a) [2 points] $f(2)$

(b) [2 points] $\lim_{x\to 0} f(x)$

(c) [2 points] $\lim_{x\to\infty} f(x)$

(d) [2 points] $\lim_{x\to 2^-} f(x)$

(e) [2 points] $\lim_{x\to 2} f(x)$

(f) [2 points] $\lim_{x\to 4} f(x)$

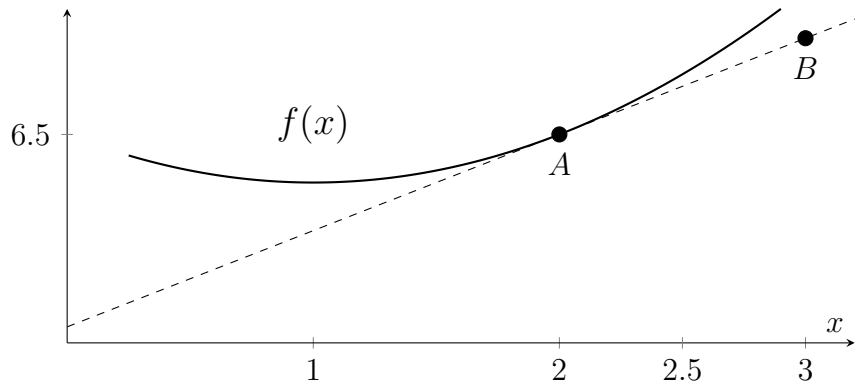
2. [6 points] Suppose $g(x) = x^2 - 1$.

   (a) [1 point] Find the value of $g(4)$.

   (b) [2 points] Simplify completely: $g(4 + h)$.

   (c) [3 points] Use your work above to **find** $g'(4)$ **using the limit definition** of the derivative.

3. [6 points] The function in the figure has $f(2) = 6.5$ and $f'(2) = 3$.



(a) [3 points] Find the formula for the tangent line to $f(x)$ at $x = 2$.

(b) [2 points] Use the picture and your equation from part (a) to find the coordinates for point B. Present your answer as an (x, y) pair.

(c) [1 point] Use your work above to estimate $f(2.5)$. Write your answer as one number.

4. [6 points] The wind speed $W(t)$ outside Madonna della Strada Chapel is measured once an hour over six consecutive hours.

Time (hours)	0	1	2	3	4	5	6
Wind (in knots)	31	21	16	13	5	7	21

(a) [3 points] Does $W'(t)$ appear to be positive or negative during the interval $[0, 3]$? Explain your answer in a sentence.

(b) [3 points] Does $W''(t)$ appear to be positive or negative during the interval $[0, 3]$? Explain your answer in a sentence.

5. [14 points] Find the requested derivatives. You are not required to simplify your final answer.

(a) [3 points] $h'(x)$ for $h(x) = \sqrt{x}(x + \sqrt{x})$

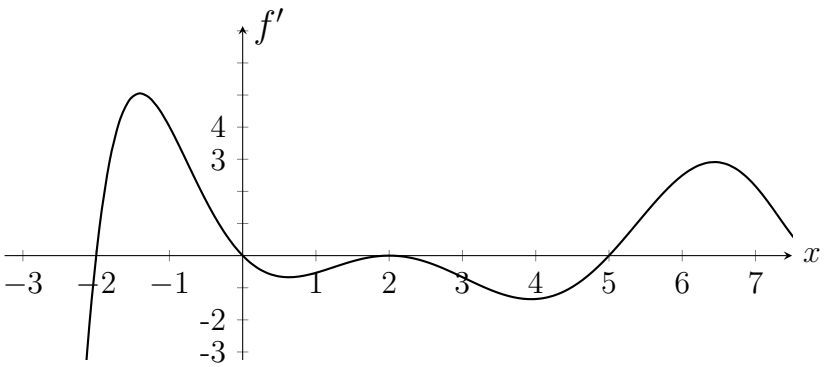
(b) [4 points] $p'(x)$ for $p(x) = 2^{\sin(x^3)}$

(c) [4 points] $q'(x)$ for $q(x) = \dfrac{\ln(x)}{x+1}$

(d) [3 points] $f''(x)$ for $f(x) = x^7 + 9x + e^{3x}$

6. [10 points] In this problem, we ask you to use information about f' to answer questions about a function f and its second derivative f''.

CAUTION: The graph below **depicts the derivative** of f.



(a) [2 points] Identify all **critical points** (x-coordinates only) for f on the interval $(-3, 7)$.

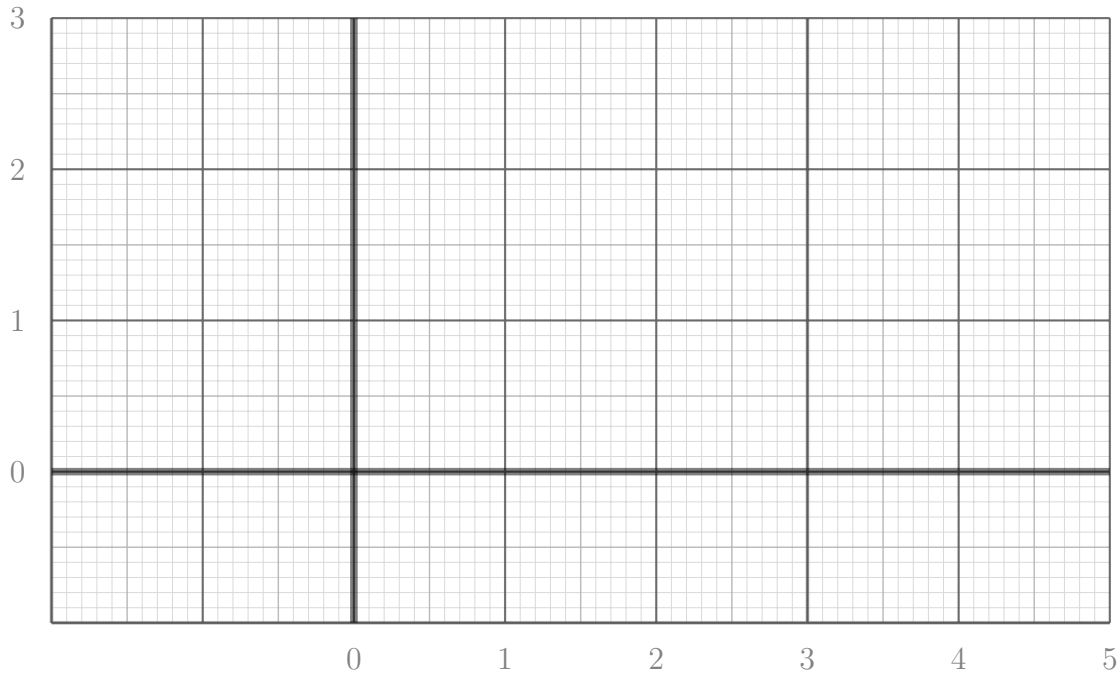
(b) [2 points] Indicate which of the above, if any, correspond to **local maxima** for f.

(c) [2 points] Which is larger, $f(6)$ or $f(7)$? (Explain.)

(d) [2 points] Which is larger, $f''(6)$ or $f''(7)$ (Explain.)

(e) [2 points] Identify all **inflection points** (x-coordinates only) for f on the interval $(-3, 7)$.

7. [4 points] Sketch the graph of a function $f(x)$ that is always decreasing, always concave up and satisfies $f(0) = 1$.



8. [10 points] A company that produces cell phones has a production capacity of up to 200 million units. The company estimates that the cost of producing a single cell phone varies with the production level and is determined by the cost function

$$C(x) = 0.05x^2 - 15x + 1625 \text{ for } 0 \leq x \leq 200,$$

where C is the cost (in dollars) of producing a single cell phone when the company has a level of production of x million cell phones.

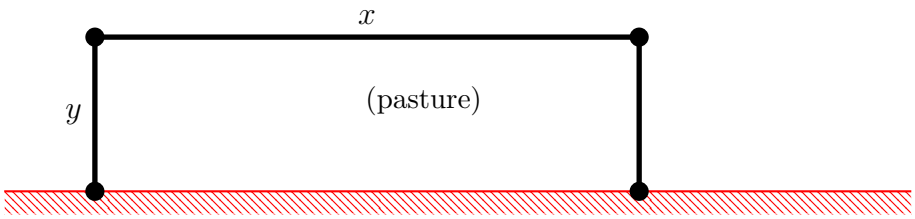
(a) [2 points] Find a formula for $C'(x)$.

(b) [3 points] Find the value of $C'(100)$ and write an interpretation for this value in the context of this problem. *Make sure to write your answer in a complete sentence and using the appropriate units.*

(c) [3 points] Find the critical points of $C(x)$ and classify them as local minima, local maxima, or neither.

(d) [2 points] What is the production level (in millions of cell phones) that gives the global minimum of the cost function C?

9. [10 points] The livestock industry has determined that, to raise healthy cattle, a farm needs 20 square yards of space per cow. A small farmer is interested in acquiring 90 cows and needs to build a rectangular pasture that only requires three sides of fencing. (They will use one side of an already existing barn).



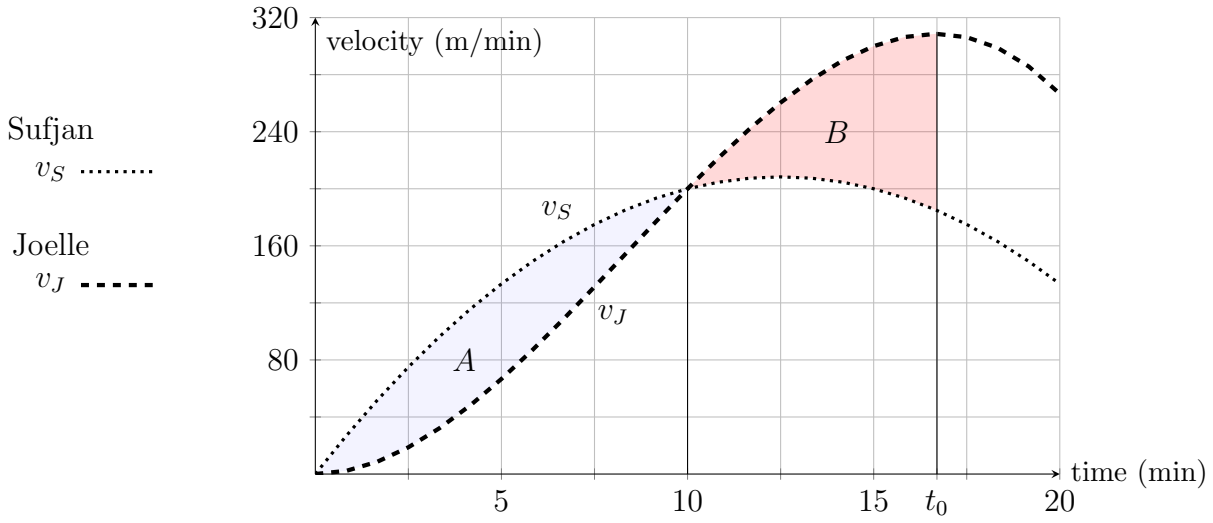
(a) [2 points] Write an equation involving x and y for the total length of new fencing (in yards) that needs to be installed to build a pasture for the cows.

(b) [3 points] Write an equation for the total length needed involving only x.

(c) [5 points] What is the minimum length of fencing that needs to be purchased to build the pasture? Include units in your answer.

10. [10 points] Sufjan and Joelle agreed to run a race for a local charity. Depicted below are the graphs of their velocities in meters per minute. (*e.g.*, 2.5 minutes after the race began, Sufjan was running at 80 meters per minute.)

    At time t_0, the **shaded regions** A and B have **equal area**.
    Suppose the winner finishes the race in 20 minutes.



(a) [2 points] True or False: Sufjan covered more ground than Joelle after 5 minutes.

(b) [2 points] True or False: Joelle caught up to Sufjan after 10 minutes.

(c) [2 points] True or False: Sufjan was A meters ahead of Joelle after 10 minutes.

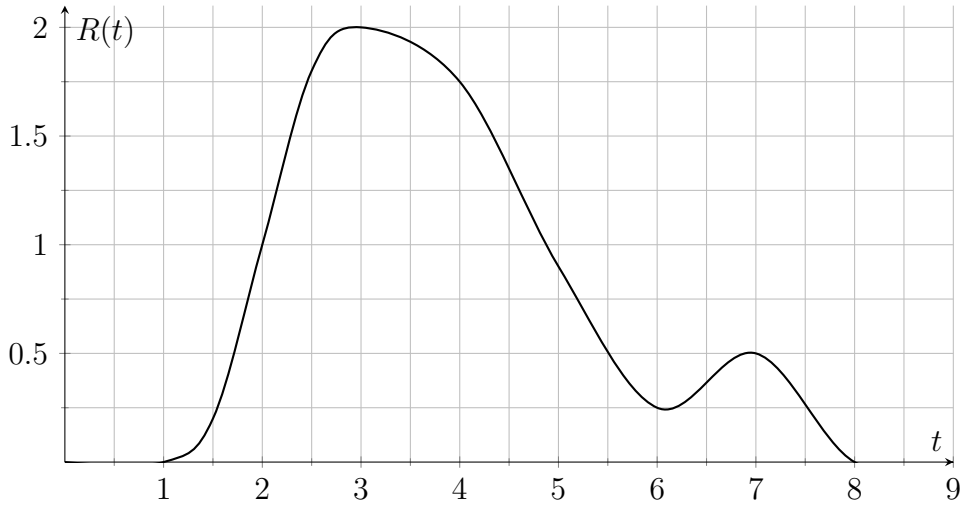
(d) [2 points] True or False: Sufjan covered more ground than Joelle after 15 minutes.

(e) [2 points] Who won the race? Justify your answer with a single sentence.

11. [10 points] (a) [5 points] Compute the indefinite integral $\int \left(x^3 + e^{3x} + \frac{1}{1+x^2} \right) dx$

(b) [5 points] Find the antiderivative $F(x)$ for $f(x) = x^3 + 6x$ that satisfies the property $F(2) = 6$.

12. [8 points] The figure below shows the rate R of snowfall (in inches per hour) during a recent winter storm in Chicago, t hours after midnight.



(a) [5 points] Estimate $\int_2^8 R(t)\,dt$ using a left Riemann sum with 3 subdivisions.

(b) [3 points] Interpret $\int_2^8 R(t)\,dt$ in the context of this question. *Make sure to write your answer in a complete sentence with units. Use the value found in* (a) *as part of your answer.*

Elementary Tools from Algebra and Geometry

Quadratic Formula: $ax^2 + bx + c = 0 \quad \rightsquigarrow \quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Pythagorean Theorem: If a right triangle has legs a, b and hypotenuse c, then $a^2 + b^2 = c^2$.

Triangle Area $= \frac{1}{2}$ base $\times$ height.

Rectangle Area = base $\times$ height

Circle Area $= \pi r^2$

Circle Perimeter $= 2\pi r$

Perimeter of a polygon (triangle, rectangle, etc.) = sum of side lengths

Five derivative rules for operations on functions.

Constant Multiple Rule: $\dfrac{d}{dx}\Big(cf(x)\Big) = cf'(x)$

Sum and Difference Rule: $\dfrac{d}{dx}\Big(f(x) \pm g(x)\Big) = f'(x) \pm g'(x)$

Product Rule: $\dfrac{d}{dx}\Big(f(x) \cdot g(x)\Big) = f'(x)g(x) + f(x)g'(x)$

Quotient Rule: $\dfrac{d}{dx}\Big(\dfrac{f(x)}{g(x)}\Big) = \dfrac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$

Chain Rule: $\dfrac{d}{dx}\Big(f(g(x))\Big) = f'(g(x)) \cdot g'(x)$

Ten derivative rules for functions

Derivative of a Constant: $\dfrac{d}{dx}(c) = 0$, where c is a constant.

The Power Rule: $\dfrac{d}{dx}\Big(x^n\Big) = nx^{n-1}$

Exponential Functions: $\dfrac{d}{dx}\Big(a^x\Big) = a^x \cdot \ln(a)$

Special Case: $\dfrac{d}{dx}\Big(e^x\Big) = e^x$

Three Trigonometric Rules:

$\dfrac{d}{dx}\Big(\sin(x)\Big) = \cos(x)$

$\dfrac{d}{dx}\Big(\cos(x)\Big) = -\sin(x)$

$\dfrac{d}{dx}\Big(\tan(x)\Big) = \sec^2(x) = \dfrac{1}{\cos^2(x)}$

Three Inverse Function Rules:

$\dfrac{d}{dx}\Big(\ln(x)\Big) = \dfrac{1}{x}$

$\dfrac{d}{dx}\Big(\arctan(x)\Big) = \dfrac{1}{1+x^2}$

$\dfrac{d}{dx}\Big(\arcsin(x)\Big) = \dfrac{1}{\sqrt{1-x^2}}$

General Antiderivative Rules

If k is a constant $\displaystyle\int k\,dx = kx + C$

$\displaystyle\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$, when $n \neq -1$

$\displaystyle\int a^x\,dx = \frac{a^x}{\ln(a)} + C$

$\displaystyle\int e^x\,dx = e^x + C$

$\displaystyle\int \cos(x)\,dx = \sin(x) + C$

$\displaystyle\int \sin(x)\,dx = -\cos(x) + C$

$\displaystyle\int \sec^2(x)\,dx = \tan(x) + C$

$\displaystyle\int \frac{1}{x}\,dx = \ln(|x|) + C$

$\displaystyle\int \frac{1}{1+x^2}\,dx = \arctan(x) + C$

$\displaystyle\int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin(x) + C$